Stability of a tunnel face in rocks using the Hoek-Brown failure criterion

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ABSTRACT

The collapse of the tunnel face could occur if the support pressure is lower than a limit value called the “critical” or “collapse” pressure. In this work, we employ an advanced rotational tunnel face failure mechanism in the context of limit analysis conducted for tunnels constructed in low quality rock masses with a Hoek-Brown (HB) failure criterion. The non-linearity of the HB criterion, however, introduces the need for additional assumptions about the distribution of normal stresses, and we employ numerical analyses to identify a constant stress distribution as an adequate initial approximation. Test cases are employed to compare the results of limit analyses with results of a three-dimensional numerical model developed to study tunnel face stability. Our results show that (i) values of the face critical pressures computed with limit analysis are similar to those obtained with the numerical model, and also (ii) that the failure mechanisms assumed in the limit-analysis approach is very similar to that computed with the numerical simulation, hence suggesting that the proposed limit-analysis approach can be employed to develop fast estimates of critical pressures for tunnel face stability.

1 INTRODUCTION

Limit analysis has been employed to compute upper and lower bounds to failure loads in a wide range of geotechnical engineering problems. In the context of tunnel excavation under a pressurized shield, for instance, the collapse of the tunnel face could occur if the support pressure is lower than a limit value called the “critical” or “collapse” pressure. To study this problem, Davis *et al*. (1980) proposed a first solution for the case of cohesive materials, whereas Leca & Panet (1988) and Leca & Dormieux (1990) proposed upper and lower bound solutions for cohesive-frictional materials. Recent research has been focused on the development of improved failure geometries (see e.g., Oberlé, 1996; Soubra, 2000), which have lead to “generalized” failure surfaces that are developed iteratively (i.e., point-to-point) and that allow rotational failure modes that affect the whole excavation front (Mollon *et al.*, 2010, 2011a).

In such works, however, failure has been modelled using the linear Mohr-Coulomb failure criterion traditionally applied to c-φ soils. The use of a non-linear failure criterion in the context of limit-analysis introduces difficulties, since the friction angle (and hence the geometry of the failure surface in an associated-flow framework) are dependent on the stress state at failure. One approach to consider non-linear failure criteria in the limit-analysis literature has been to employ a linear failure envelope that is tangent to the original (and non-linear) failure criterion (Yang & Yin, 2004), since upper-bound solutions obtained using such envelope are also upper-bound solutions to the original problem with the non-linear criterion (Chen, 1975). For instance, the “generalized tangential technique” (Yang *et al.*, 2004a,b) is based on substituting the non-linear failure criterion by a linear envelope that is tangent at an "optimum" point. Huang & Yang (2010) have used this technique to study the stability of the tunnel face employing the passive mechanism of Leca & Dormiuex (1990).

In this work, we study the problem of tunnel face stability using limit-analysis and the Hoek-Brown (HB) failure criterion that is typically applied to fractured rock masses (Hoek *et al.*, 2002). The HB criterion is a non-linear criterion that can be expressed as:

$$\sigma_1' = \sigma_3' + \sigma_{ci} \cdot \left(m_b \cdot \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a, \tag{1}$$

where σ_1' and σ_3' are the major and minor effective principal stresses; σ_{ci} the UCS of the intact rock; s and a are parameters that depend of rock mass quality; and m_b is a parameter that depends on rock mass quality and rock type.

2 A ROTATIONAL FAILURE MECHANISM FOR TUNNEL FACES IN HB MATERIALS

In this work we generalize the single-block rotational failure mechanism proposed by Mollon *et al.* (2011a) so it can be employed with the HB failure criterion. In the original formulation (refer to Mollon *et al.* (2011a) for details), the geometry of the failure mechanism is defined "point-by-point" using a centre of rotation (whose location needs to be optimized) and the condition of associated flow rule, so that the angle formed by the velocity vector and the failure surface is constant in this case.

To be able to use the (non-linear) HB failure criterion in the definition of the generalized failure geometry, however, we need to express the HB criterion in terms of the shear (τ) and normal (σ_n) stresses at the failure plane. A solution to such problem has been proposed by Kumar (1998), who showed that the failure envelope for the generalized HB criterion (i.e., with $a \neq 0.5$; for a solution to the $a = 0.5$ case, see also (Ucar, 1986)) could be defined using the following parametric equations:

$$\frac{\sigma_n}{\sigma_{ci}} = \frac{1}{m_b} \cdot \left(\frac{m_b a}{2}\right)^{\frac{1}{1-a}} \cdot \left(\frac{1-\sin\beta}{\sin\beta}\right)^{\frac{1}{1-a}} \cdot \left(1 + \frac{\sin\beta}{a}\right) - \frac{s}{m_b} \tag{2}$$

$$2 \cdot \frac{\tau}{\sigma_{ci}} = \frac{\cos\beta}{\left(1+\frac{\sin\beta}{a}\right)^a} \cdot \left(m_b \cdot \frac{\sigma_n}{\sigma_{ci}} + s\right)^a, \tag{3}$$

where β is the instantaneous friction angle. Using Eqs. (2) and (3), we can define, for each value of σ_n, the tangent to the failure envelope and, therefore, "equivalent" values of cohesion and friction angle. Such parameters can then be employed for the construction of a failure mechanism that follows the assumption of associated flow rule.

However, to perform the linearization indicated above, we need the stresses acting along the failure surface. Since such stresses are not known in general (see below for further discussion), we need to introduce additional parameters that define the stress distribution along the failure surface so that, optimizing in terms of such variables (i.e., parameters that define the rotation centre location and the stress distribution), we can achieve an "optimum" solution to the upper-bound limit analysis problem.

3 STRESS DISTRIBUTION AT THE TUNNEL FACE

As indicated above, we need to define the distribution of normal stresses along the failure surface. Theoretically, we could aim for a very flexible (i.e., with many degrees of freedom) distribution that would introduce no constraints in the quality of the solution obtained; this approach, however, has the shortcoming that it increases the dimensionality (and, hence, the difficulty) of the optimization problem.

We employed FLAC^{3D} to conduct numerical simulations that help us identify reasonable (yet simple) stress distributions at the tunnel face and, in particular, to compute the distributions of normal stresses along the failure surface (the details of the numerical model are discussed below). Figure 1 shows the obtained results, with Fig. 1(a) showing spatial distribution of normal stresses, and with Fig. 1(b) showing the stress values along the vertical plane of symmetry of the tunnel.

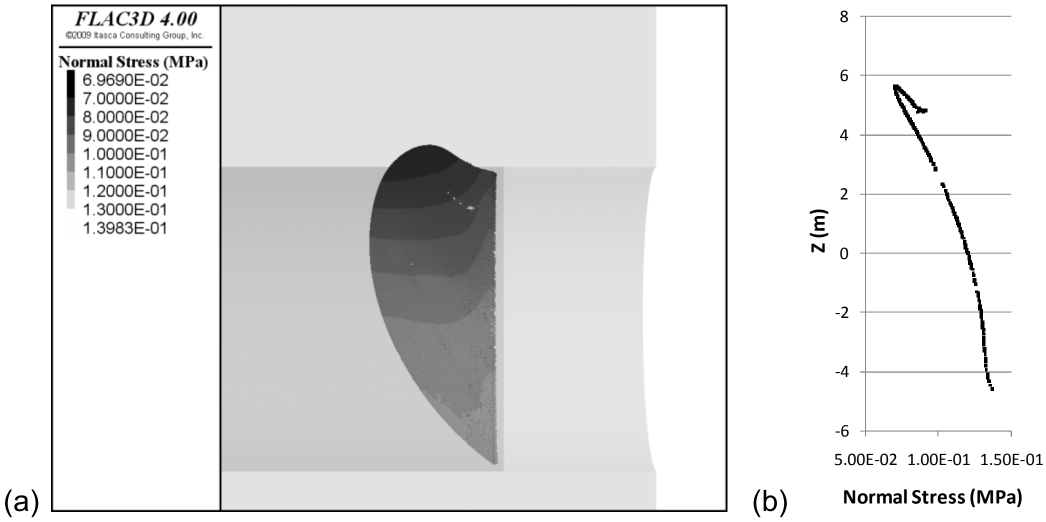

Figure 1 Distribution of normal stresses along the failure surface: (a) spatial distribution and (b) along the vertical plane of symmetry

As it can be observed, the normal stress values along the failure surface tend to increase with depth. Based on such results, in this work we choose to work with a uniform distribution of normal stresses. (Research is currently underway to test other types of stress distributions -linear, etc.-, although results are not yet complete and will be presented elsewhere). Such selection is motivated by ease of optimization (only one parameter is needed to define the distribution) and by the observation that the stress distribution is close-to-uniform in the lower part of the section, which is the most critical zone for the tunnel face stability (Mollon *et al.*, 2011b).

4 COMPUTED RESULTS

4.1 Definition of test-cases

To test the proposed methodology, we have used two test-cases to compare results obtained with our methodology with results obtained using numerical models in FLAC^{3D}.

Table 1 presents a list with parameters employed for the analyses of the two test-cases considered. Parameters m_b, s, and a needed for the HB criterion (see Eq. (1)) have been computed using the latest available version of the HB criterion (Hoek *et al.*, 2002), and a damage parameter of D = 0 has been employed in both cases to represent TBM-excavated tunnels. As shown in Table 1, cases have been selected so that they correspond to low-quality rock masses ($GSI \leq 25$), since it is in such types of rock masses where problems associated to tunnel face instability could be mainly expected in real practice.

Table 1 Parameters employed in the test-cases considered for validation

Case	m_i	σ_{ci} (MPa)	GSI	γ (t/m^3)
I	9.6	5	20	2.5
II	4	3	10	2.5

4.2 Description of the numerical model

Figure 2 shows an illustration of the FLAC^{3D} model employed for numerical simulation of tunnel face stability. The tunnel considered has a diameter of D = 10 m and a cover of C = 20 m. To minimize boundary effects and the needed computational effort, the tunnel has been represented using a symmetric model with dimensions (in meters) of 25x30x35. A total of 163.260 elements have been employed, of which 819 are in the tunnel front. (Note also that the meshing has been designed to minimize element sizes where larger stress gradients are expected). The boundary conditions of the model are given by fixed displacements at the "artificial" boundaries of the model (i.e., at its lateral perimeter and at its base); similarly, and since we are only concerned with face stability (and not with tunnel convergences), the tunnel support has not been included and displacements at the tunnel excavation boundary have also been fixed. The constitutive model employed for the fractured rock mass is elastic-perfectly plastic with the HB failure criterion. The model has been considered with associated flow rule to have the same conditions as the limit-analysis approach although, in this case, that aspect does not seem to have much influence on our computed results.

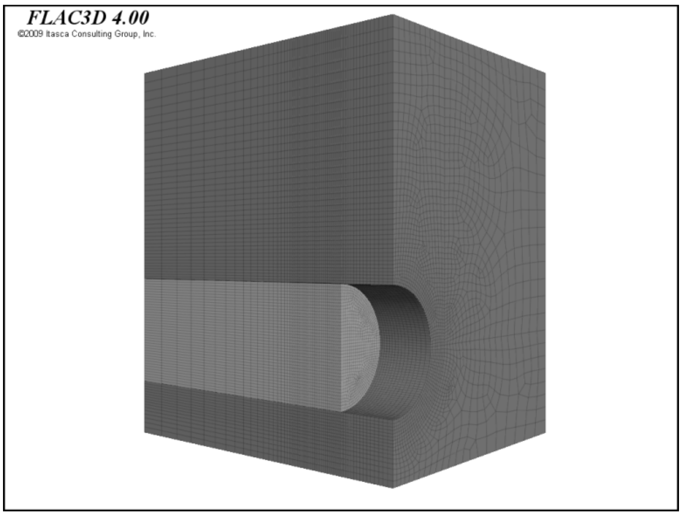

Figure 2 Geometry of the developed numerical model

To find the collapse pressure at the tunnel face, the bisection method proposed by Mollon *et al.* (2009) has been employed. Using this method, and for a given interval of pressure values (defined by one higher value for which the face is stable and by one lower value for which it is unstable), the stability of the face is computed assuming the mean value; if the face is stable, the higher interval boundary is substituted by such mean value (the lower interval value is substituted if unstable), and the process is iteratively repeated for such newly defined intervals until the required precision is achieved, which in this case has been set to 0,1 kPa. As a convergence criterion, the FLAC^{3D} default value of 1EXP-05 for the unbalanced mechanical-force ratio has been tightened to 1EXP-07.

4.3 Comparison of results

To check the results of our proposed limit-analysis approach with respect to the results of numerical simulations, we have compared (i) the computed values of the collapse pressure at the tunnel face; and (ii) the shapes of the failure mechanisms obtained by both methods. (The failure mechanism in the FLAC^{3D} model has been estimated considering the distribution of shear deformations).

Table 2 presents a comparison of collapse face pressures computed using the numerical model and also of results computed using our limit analysis approach with the HB criterion and a constant stress distribution along the failure surface. Similarly, Figure 3 presents a comparison of the failure mechanisms obtained (for both Cases I and II) with our limit-analysis approach and with the FLAC^{3D} numerical simulations.

Table 2 Comparison of results

Case	Collapse face pressure (kPa)	
	Limit Analysis	Numerical Simulation
I	2.62	2.69
II	29.31	29.98

(a) (b)

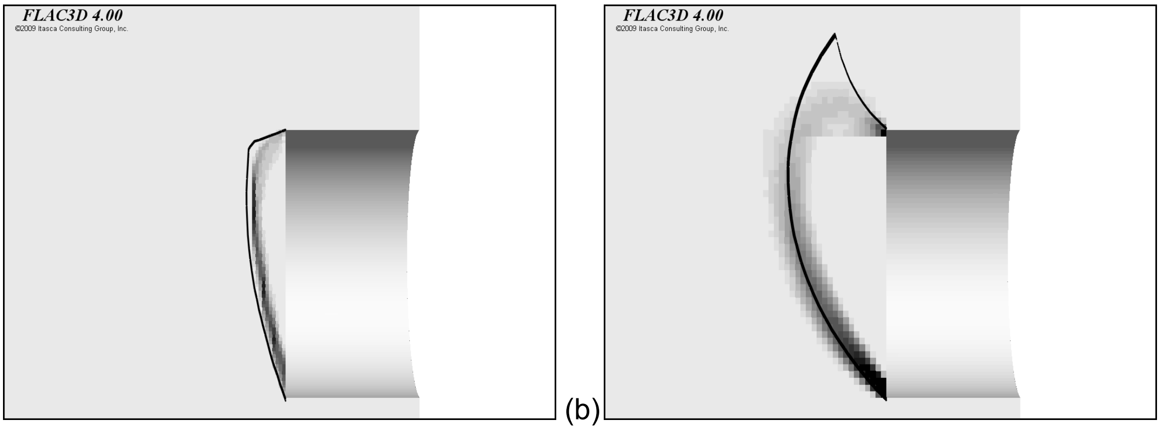

Figure 3 Comparison of failure mechanisms computed with the limit-analysis and with the numerical simulation: (a) Case I and (b) Case II

5 CONCLUSIONS

In this work we analyze the face stability of shallow tunnels excavated in a fractured rock mass with the (non-linear) Hoek-Brown failure criterion. (Note that the method is not valid for squeezing problems in over-stressed rock masses). To that end, we employ an advanced (and recently proposed) failure mechanism for the tunnel face (Mollon *et al.*, 2011a); the mechanism, that covers the whole excavation front, is generated “point-by-point”, and it allows for a rotational-type failure mechanism that is very similar to that observed in small-scale tunnel tests in the laboratory.

The use of a non-linear failure criterion introduces the need to consider the distribution of normal stresses along the failure surface, so that the instantaneous friction angle can be computed to fulfil the assumption of associated-flow that is inherent to the limit-analysis approach. Numerical simulations have been employed to identify adequate (yet simple) shapes of stress distributions at the tunnel face, and the results of such simulations suggest that a constant distribution of stresses along the failure surface could be employed as an initial approximation to the real stress distribution.

Two test-cases corresponding to rock masses with low quality (as indicated by the GSI value) have been employed to compare our results of the limit-analysis approach with results of three-dimensional simulations conducted with FLAC^{3D}. Two aspects have been compared: (i) the numerical value of the collapse pressure; and (ii) the shape of the failure mechanism. The obtained results suggest that the limit-analysis approach proposed herein could be applied for fast (and relatively reliable) estimations of the pressure needed for face support in shallow tunnels excavated in heavily fractured rock masses.

REFERENCES

Chen, W.-F. 1975. Limit analysis and soil plasticity. Amsterdam: Elsevier.

Davis, E. H., Gunn, M. J., Mair, R. J., & Seneviratine, H. N. 1980. The stability of shallow tunnels and underground openings in cohesive material. Géotechnique, 30(4): 397–416(19).

Hoek, E., Carranza-Torres, C., & Corkum, B. 2002. Hoek-brown failure criterion - 2002 edition. Proceedings of NARMS-TAC Conference, Toronto, 267-273.

Huang, F., & Yang X.-L. 2010. Upper bound solutions for the face stability of shallow circular tunnels subjected to nonlinear failure criterion. Proceedings of the 2010 GeoShanghai International Conference.

Kumar, P. 1998. Shear failure envelope of Hoek-Brown criterion for rockmass. Tunnelling and Underground Space Technology, 13(4): 453–458.

Leca, E., & Panet, M. (1988). Application du calcul à la rupture à la stabilité du front de taille d'un tunnel. Revue Française De Géotechnique, (43), 5-19.

Leca, E., & Dormieux, L. 1990. Upper and lower bound solutions for the face stability of shallow circular tunnels in frictional material. Géotechnique, 40(4): 581–606(25).

Mollon, G., Dias, D., & Soubra, A.-H. 2009. Probabilistic analysis of circular tunnels in homogeneous soil using response surface methodology. Journal of Geotechnical and Geoenvironmental Engineering (ASCE), 135(9): 1314-1325.

Mollon, G., Dias, D., & Soubra, A.-H. 2010. Face stability analysis of circular tunnels driven by a pressurized shield. Journal of Geotechnical and Geoenvironmental Engineering (ASCE), 136(1): 215–229.

Mollon, G., Dias, D., & Soubra, A.-H. 2011a. Rotational failure mechanisms for the face stability analysis of tunnels driven by a pressurized shield. International Journal for Numerical and Analytical Methods in Geomechanics, 35(12): 1363–1388.

Mollon, G., Phoon, K. K., Dias, D., & Soubra, A.-H. 2011b. Validation of a new 2D failure mechanism for the stability analysis of a pressurized tunnel face in a spatially varying sand. Journal of Engineering Mechanics, 137(1): 8–21.

Oberlé, S. 1996. Application de la méthode cinématique á l'étude de la stabilité d'un front de taille de tunnel. France: Final project, ENSAIS.

Soubra, A.-H. 2000. Three-dimensional face stability analysis of shallow circular tunnels. Proceedings of the International Conference on Geotechnical and Geological Engineering, 19-24 November, 2000.

Ucar, R. 1986. Determination of shear failure envelope in rock masses. Journal of Geotechnical Engineering (ASCE), 112(3): 303–315.

Yang, X.-L., Li, L., & Yin, J.-H. 2004a. Stability analysis of rock slopes with a modified Hoek-Brown failure criterion. International Journal for Numerical and Analytical Methods in Geomechanics, 28(2): 181–190.

Yang, X.-L., Yin, J.-H., & Li, L. 2004b. Seismic and static stability analyisis for rock slopes by a kinematical approach. Géotechnique, 54(8): 543–549.

Yang, X.-L., & Yin, J.-H. 2004. Slope stability analysis with nonlinear failure criterion. Journal of Engineering Mechanics, 130(3): 267–273.