A new 2D failure mechanism for face stability analysis of a pressurized tunnel in spatially variable sands

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ABSTRACT

This paper presents a method to consider the spatial variability of the soil shear strength parameters for determining the critical collapse pressure of a pressurized tunnel face. Only the case of a cohesionless soil is considered in the analysis. The present method is based on the kinematic theorem of limit analysis. A new 2D kinematically admissible collapse mechanism whose shape depends on the spatial distribution of the soil friction angle (φ) is proposed. In this mechanism, the normality condition imposed by limit analysis is enforced everywhere along the slip surfaces of the failure mechanism. The results obtained using the present approach are presented and compared to those based on common numerical methods such as the Finite Element Method (FEM) or the Finite Difference Method (FDM). The proposed method is computationally more efficient and has significant potential for simulation studies involving random fields.

INTRODUCTION

Stability is a key design/construction consideration in real shield tunnelling projects. The aim of stability analysis is to ensure safety against soil collapse in front of the tunnel face. This requires determination of the minimal pressure (air, slurry or earth) required to prevent the collapse of the tunnel face. Face stability of circular tunnels driven by pressurized shields has been investigated by several authors in the literature. Some authors have performed experimental tests (cf. Chambon and Corté 1994 and Takano et al. 2006). Others (Horn 1961, Leca and Dormieux 1990, Eisenstein and Ezzeldine 1994, Broere 1998, Mollon et al. 2009a and b, Mollon et al. 2010) have studied the problem using analytical or numerical approaches. Fig. 1a presents the geometry of the problem considered in this paper. This study focuses on pressurized shields using compressed air as the retaining fluid. As a result, the applied face pressure σ_t is uniform. If this pressure drops below a critical value (critical collapse pressure σ_c), the soil mass abutting the tunnel face can collapse into the tunnel. In the case of a cohesionless or a frictional and cohesive soil, the arch effect can prevent this failure from reaching the ground surface especially if the cover depth C is large enough with respect to the diameter D of the tunnel. In this study, for the sake of simplicity, it will be assumed that the soil is cohesionless and that the failure mechanism never outcrops, which is always true in practice as long as C>D (Mollon et al. 2010). The present work is undertaken as part of a broader objective to study the impact of a spatially varying soil on the value of σ_c. Realizations of spatially varying soil can be generated quite readily using existing algorithms such as the Karhunen-Loeve method and a Monte Carlo sampling method. Though reliable, this method requires a large number of calls of the deterministic model which can be very time-consuming if one uses common numerical methods such as FEM or FDM. The present paper proposes an alternative and more efficient method: a kinematical approach in limit analysis. A failure mechanism that is able to deal with spatial variations of the soil shear strength parameters is proposed. This mechanism presents a main advantage for downstream stochastic analysis: it is much less time-consuming than common numerical methods such as FEM or FDM. However, this model has to be validated before using it in an extensive Monte Carlo simulation. This is done here by introducing some artificial weaknesses (called "pixels") of several sizes and shapes *systematically* at several locations in the soil mass. The impact of these weak "pixels" on the critical collapse pressure is studied. The same cases are treated with the commercial numerical software FLAC^{3D} (1993) in order to check that the proposed mechanism correctly accounts for these local strength weaknesses. The proposed mechanism can be applied to a general c-φ soil but only results for cohesionless soils are presented due to page length limitations.

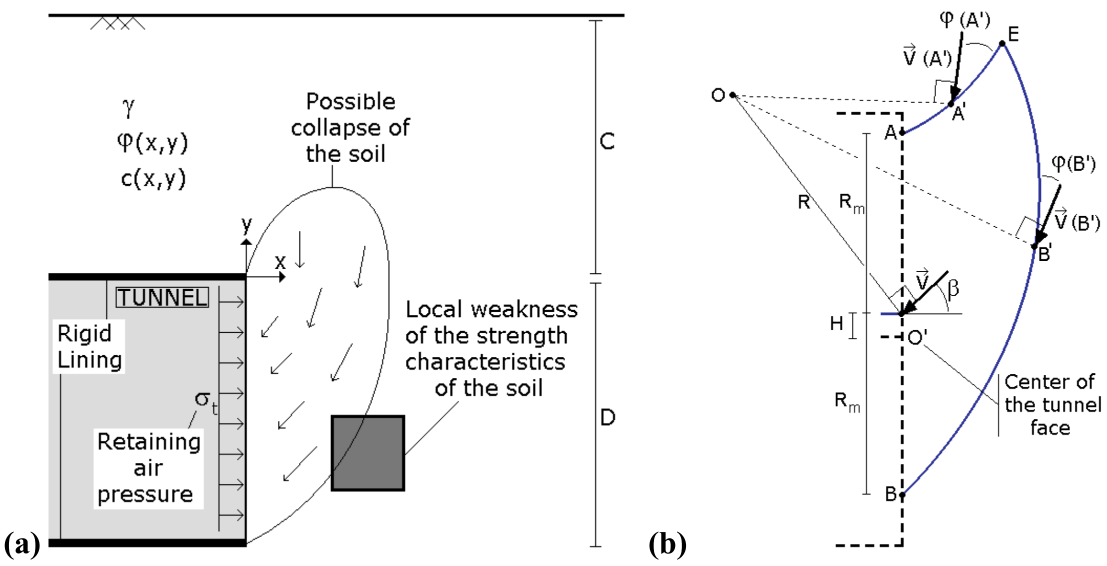


Fig. 1: (a) Geometry of the problem and (b) definition of the four parameters governing the proposed failure mechanism (R, β, H and R_m)

PROPOSED COLLAPSE MECHANISM

The limit analysis failure mechanism should be able to take account of the spatial variations of the soil shear strength properties and to provide a good approximation of the critical collapse pressure. This implies that the failure mechanism should not be too different from the physical failures that can be observed during real tunnel face collapses. Several authors undertook studies of tunnel face collapse in cohesionless soils using reduced models under 1g [Takano et al. (2008)] and in centrifuge [Chambon and Corté (1996)]. They reported that the failure pattern can be modelled by a rotational rigid-block mechanism and that the failure surfaces can be approximated by logarithmic spirals. This is in good agreement with the theory of limit analysis which states that a rotational failure mechanism in a soil leads to log-spiral slip surfaces. Notice that all the existing failure mechanisms considered for the analysis of different stability problems in geotechnical engineering were created to deal with homogeneous soils. To our best knowledge, the mechanism in the presence of spatial variations of the soil shear strength parameters has not been studied yet. This problem is being addressed by the proposed 2D mechanism described below. The assumptions adopted in the analysis are listed below:

- The soil is assumed to be an associated flow rule Coulomb material. Only the case of a cohesionless soil is considered herein;
- The assumed failure mechanism is a rotational rigid-block mechanism;
- The failure mechanism is kinematically admissible which implies that the normality condition must be enforced at all the velocity discontinuity surfaces;
- The 2D mechanism intersects the tunnel face in two points A and B as shown in Fig. 1b.

The failure mechanism is described by four parameters as shown in Fig. 1b: R and β are related to the position of the centre O of rotation, and H and R_m are related to the position of the two points A and B where the mechanism intersects the tunnel face. H and R_m are introduced to cater to the possibility of local face collapse in the presence of strength heterogeneities. In a homogeneous soil, these parameters would be inapplicable because the mechanism would always intersect the whole tunnel face.

The normality condition requires that the slip surface always makes an angle φ with the velocity vector $\vec{V}$. If the friction angle is spatially varying, then the slip surface at any point with coordinates (x, y) should make an angle φ(x, y) with the velocity vector $\vec{V}(x, y)$. As shown in Fig. 1b, this implies that the two slip surfaces emerging from A and B are concave with respect to point O, and that these two lines will eventually meet in a point E which is the extremity of the mechanism. For a homogeneous soil, the normality condition would lead (for a velocity field described by a rigid-block rotation about point O) to a rigid block delimited by two logarithmic spirals. These two logarithmic spirals come from points A and B respectively and terminate at point E which constitutes the extremity of the moving block. Such a mechanism would be easy to define analytically but would not be able to deal with a spatial variation of the friction angle of the soil (φ). To address this problem, an angular discretization scheme is adopted for the two curves emerging from A and B (Fig. 2).

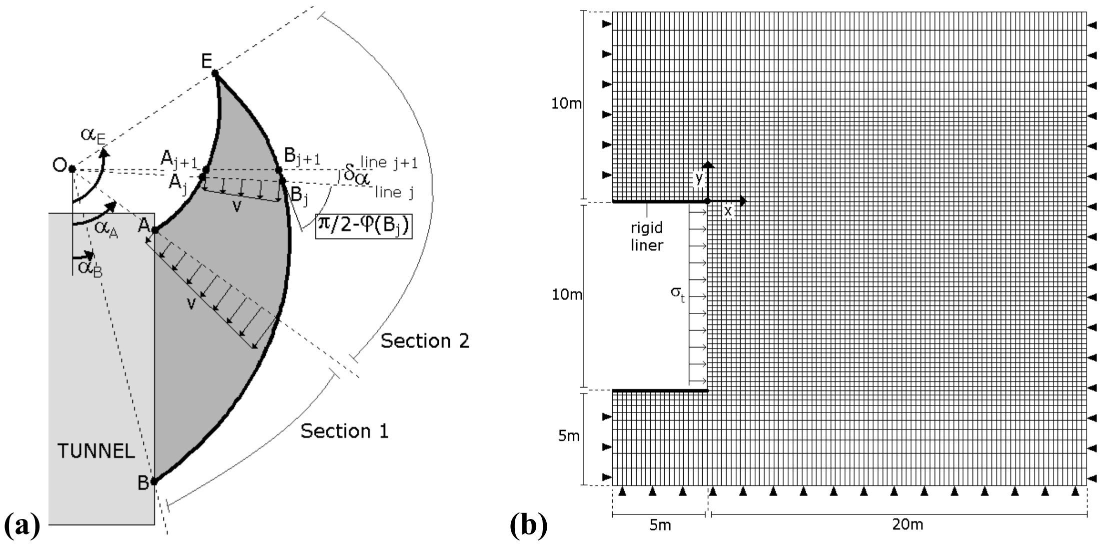


Fig. 2: (a) Principle of generation of the proposed limit analysis model and (b) mesh of FLAC numerical model

The generation process uses several radial lines meeting at the point of rotation O. The angle between two successive lines is equal to a user-specified constant value δ_α. These lines are defined by an index j with j=0 corresponding to line OB. This set of sweeping lines with a common origin O terminates at line OE (unknown a priori). The mechanism is divided into two sections. As shown in Fig. 2, Section 1 includes only the lower slip line, and Section 2 includes two slip lines (the upper and the lower one). The generation process aims at defining a collection of points belonging to the slip lines. Both points A_{j+1} et B_{j+1} belonging to line j+1 are defined from previous points A_j et B_j belonging to line j, the initial points being A and B for the upper and lower slip lines respectively. For example, point B_{j+1} can be deduced from point B_j using the following conditions:

- B_{j+1} belongs to line j+1
- The straight segment B_jB_{j+1} makes an angle $\varphi(B_j)$ with the velocity vector $\vec{v}(B_j)$ at point B_j. Note that this velocity is normal to line j due to the rigid block rotation.

For a spatially varying soil, the angle φ to be considered at point B_j is the local value of the friction angle at point B_j, called $\varphi(B_j)$. It is then straightforward to generate all the points B_j starting from B until the end of Section 1. For Section 2, the same method is used to generate points A_{j+1} and B_{j+1} from points A_j and B_j respectively. The upper and lower slip lines are generated until they cross at point E.

With this generation process, the mechanism is constrained to respect the normality condition at each point A_j and B_j of its contour. The determination of the collapse pressure corresponding to this mechanism is based on the work equation, which states that the rate of work of the external forces applied to the moving soil mass is equal to the rate of energy dissipation. The forces applied to the moving block are: (i) the weight of the soil composing the moving block, (ii) the collapse face pressure, and (iii) a possible surcharge on the ground surface in case of outcrop of the mechanism (not considered in the present study). The energy dissipation takes place at the velocity discontinuity surfaces in case of failure mechanisms involving rigid block movement, and is proportional to the cohesion. Since the cohesion is set to zero in the present paper, the energy dissipation is null. Finally, the work equation in the case of a cohesionless soil with no surcharge loading on the ground surface becomes:

$$\dot{W}_\gamma + \dot{W}_T = 0 \tag{1}$$

$$\dot{W}_T = \iint_\Sigma \overrightarrow{\sigma_u} \cdot \vec{v} \cdot d\Sigma \tag{2}$$

$$\dot{W}_\gamma = \iiint_V \vec{\gamma} \cdot \vec{v} \cdot dV \tag{3}$$

The expressions (2) and (3) are calculated numerically. From Eq. (1), one obtains:

$$\sigma_u = \gamma \cdot D \cdot N_\gamma \tag{4}$$

N_γ is a dimensionless parameter which depends on the size and shape of the mechanism (and consequently on the spatial distribution of the friction angle) and on the four geometrical parameters describing the mechanism (Fig. 1b). The best (i.e. highest) solution that the mechanism can provide is obtained by maximisation of σ_u with respect to the four parameters, and is called σ_c:

$$\sigma_c = \max_{R,\beta,H,R_m} (\sigma_u) \tag{5}$$

If the soil is homogeneous, then the maximum is unique and the maximisation of σ_u can be done with a classical optimisation algorithm such as the optimisation tool implemented in Matlab. The resulting failure surfaces would reduce to two logarithmic spirals. With a heterogeneous soil, it was observed that several local maximums of the collapse pressure may exist. To locate the global maximum, an exhaustive search over the full range of the four parameters was performed. The search for the global maximum was carried out in two steps. First, a coarse search was done by evaluating σ_u at all combinations of the parameters taking the following ranges: $15° \le \beta \le 60°$ (step: 2.5°); $3m \le R \le 20m$ (step: 1m); $-1.5m \le H \le 1.5m$ (step: 0.5m); $3.5m \le R_m \le 5m$ (step: 0.5m). In a second step, the maximum found by this coarse grid was refined by reducing the steps on the four parameters. The global maximum was finally determined with the following precision for the four parameters: 0.5° for β, 0.2m for R, 0.25m for H, and 0.1m for R_m. The two steps of the optimization process require about 5000 calls of the model. More efficient optimization techniques such as the genetic algorithm would be studied in the future.

DESCRIPTION OF THE NUMERICAL MODEL

The simulations presented in this study make use of the FLAC model shown in Fig. 2b. The two vertical and the lower horizontal boundaries are assumed to be fixed in the normal direction. The dimensions of the model were adopted to ensure that the boundaries do not affect the critical collapse pressure (not detailed here). The model is composed of 7800 zones ("zone" is a FLAC terminology for a discretized element) and 16000 grid-points. The tunnel face is divided vertically into 40 zones.

The upper and lower lining of the tunnel are modelled as linear elastic. Their elastic properties are: Young's modulus $E = 15\,GPa$ and Poisson's ratio $\nu = 0.2$. The lining is connected to the soil *via* interface elements that follow Coulomb's failure criterion. The interface is assumed to have a friction angle equal to two thirds of the soil angle of internal friction and no cohesion. Normal stiffness $K_n = 10^{11}\,Pa/m$ and shear stiffness $K_s = 10^{11}\,Pa/m$ were assigned to this interface. These parameters are functions of the neighbouring elements rigidity (FLAC^{3D}, 1993). The elastic and plastic parameters of the interface were found not to have a significant influence on the collapse pressure. The soil is assigned a perfect elastic-plastic constitutive model with E=240MPa and ν=0.22. These elastic properties do not have any significant impact on the critical collapse pressure. For this reason, a very high value of E was chosen because it increases the computation speed. Concerning the values of the soil shear strength parameters used in this paper, they will be given in the next sections.

The fastest method for determining the collapse pressure would be a strain-controlled method. This method is not adequate in the framework of the stability analysis of tunnels because it implies that deflected shape of the tunnel face is known. This shape is not known *a priori* and any assumption (such as uniform or parabolic deflection) may lead to errors in the determination of the collapse pressure. The only other possibility is a stress-controlled method which consists of applying a prescribed uniform value of σ_t and testing whether or not the tunnel face is stable. As expected, the determination of a satisfactory value of the critical collapse pressure requires a great number of runs of such a simulation which is very time-consuming. A rational bisection method was coded in the FLAC software language (called FISH). It allows the critical collapse pressure of the tunnel face to be determined with an accuracy of 0.1kPa. The maximal unbalanced force ratio adopted in the paper is equal to 10^{-7}. This value is smaller than the one imposed in FLAC and was found necessary to obtain optimal results.

NUMERICAL RESULTS FOR HOMOGENEOUS SOIL

The validity of the proposed mechanism is evaluated in this section over the whole range of typical friction angles for sands (i.e. from 30° to 45°) considering a tunnel with D=10m and a soil with γ=18kN/m^3.

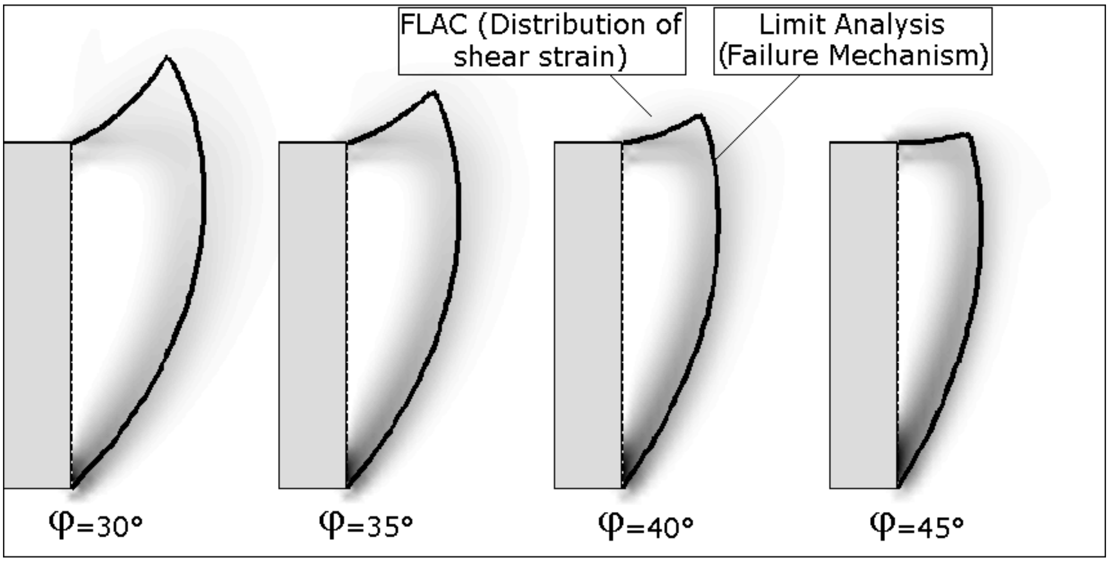
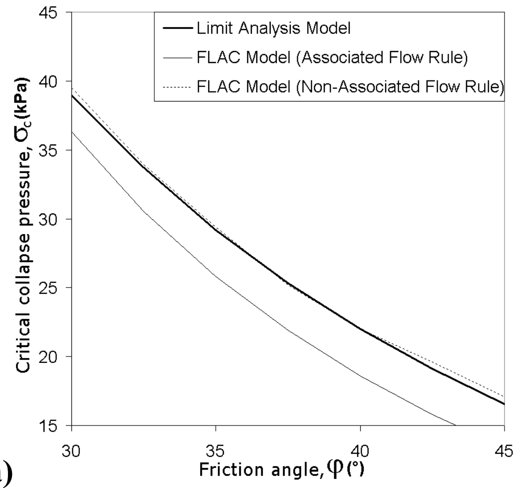


Fig. 3: Comparison of the failure mechanism provided by the proposed model and the shear strain distributions provided by FLAC for an associated flow rule

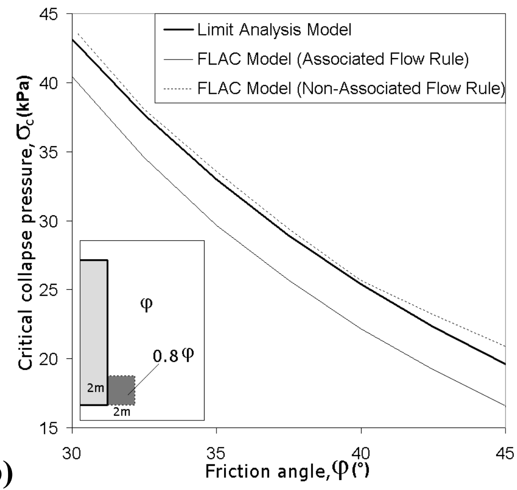
The results obtained in the case of a homogeneous soil are plotted in Figs. 3 and 4a. The discretization parameter for the limit analysis mechanism is δ_α=1°. The computation time was about 3 minutes for the limit analysis model and 120 minutes for the FLAC numerical model, both on a Core2 Quad CPU 2.40GHz. This illustrates the claim that the proposed limit analysis model is much more efficient concerning the computation time and hence more practical for stochastic simulation.

Fig. 3 shows the most critical position of the slip lines provided by the limit analysis mechanism as well as the plastic shear strain in the soil provided by FLAC simulations, in the case of a purely frictional soil and for several values of φ. There is a reasonably good agreement between the two approaches.

Fig. 4a presents the σ_c values provided by limit analysis and FLAC model in the case of a homogeneous soil for different φ values and for two values of the dilatancy angle (ψ=φ and ψ=0). It is believed that the plastic behaviour of a real sand is somewhere between these two limit values of the dilatancy angle. The curves show good qualitative agreement in terms of the trend. However, an anomaly should be pointed out. Though the proposed kinematical approach is known to provide a rigorous solution and this solution is expected to be lower (this is because the tunnel pressure resists collapse) than the exact one in the framework of limit analysis, the present mechanism was shown (Fig. 4a) to give higher values of the pressure than the numerical model for ψ=φ (associated flow rule). A thorough analysis of FLAC simulations (not shown in this paper) has shown that FLAC solutions can become very close to the limit analysis solutions for an extremely fine mesh. This may be related to the fact that a coarse mesh is not able to deal correctly with high velocity gradients. The resulting computation time of this refined model is equal to three days, which is obviously too long for a practical use. Thus, the agreement between both solutions is believed to be sufficient to validate the proposed mechanism in homogeneous sand. As a conclusion, the limit analysis model is more efficient for future use in a probabilistic context.



(a)



(b)

Fig. 4: Critical collapse pressures provided by the proposed mechanism and FLAC model (with ψ=φ and ψ=0), for (a) a homogeneous sand and, (b) a local decrease of φ by 20% in a 2m square pixel at the tunnel invert

IMPACT OF A LOCAL WEAKNESS IN THE SOIL

Fig. 4b presents the study of the impact of a local weakness in the soil on the critical collapse pressure. This is done by introducing a so-called weak "pixel" in the soil mass. A weak square pixel (with a reduction of φ by 20%) has been introduced at the invert of the tunnel face (see graphic insert). Notice that the time cost of the limit analysis model is not increased by the introduction of the local weakness. The comparison between limit analysis and FLAC results has shown a similar trend to the homogeneous case. The same interpretation given above applies in the present case. Notice that the collapse pressures provided by limit analysis and FLAC models in the presence of a local weakness are higher than those of the homogeneous case. Finally, notice that the case of a reduction of 20% of φ in this special pixel, for an initial value of φ=30°, will be designated as the "reference case" for subsequent comparisons.

Considering the impact of a single weak pixel in the soil mass, the critical collapse pressure will potentially depend on: (i) the location of the pixel, (ii) the reduction of friction angle in the weak pixel, (iii) the size of the pixel, (iv) the shape of the pixel (e.g. square, rectangle, layer…), (v) the orientation of the pixel (horizontal, vertical, inclined), and (vi) the effect of neighbouring pixels. Fig. 5a shows the impact of the elevation (y-coordinate) of a 2m × 2m square pixel (with a reduction of φ by 20%). This pixel is located just behind the tunnel face. It appears that the most critical location of the pixel is at the invert of the tunnel which corresponds to our reference case (σ_c increases in this case by 11% with respect to the homogeneous case), and the second most critical location is at its crown (increase of σ_c by 3%). The pixels in the middle of the face seem to have almost no impact on the tunnel face stability. The location of the weak pixel is therefore quite influential. The proposed mechanism compares reasonably well with FLAC results for both ψ=φ and ψ=0.

Fig. 5b shows the relative increase in σ_c with the local decrease of the friction angle for several sizes of the weak pixel (cases A, B, and C shown in the graphic insert). The critical pressure increases linearly with the local decrease of φ, and the rate of change compares favourably with that from FLAC. The size of the pixel appears to have a considerable impact on the relative increase of σ_c. This increase is larger for a given reduction of φ when the size of the pixel increases. For example, the cases B and C of 4m and 6m square pixels respectively, produce larger increase of σ_c than the 2m square pixel reference case, for the same reduction of φ.

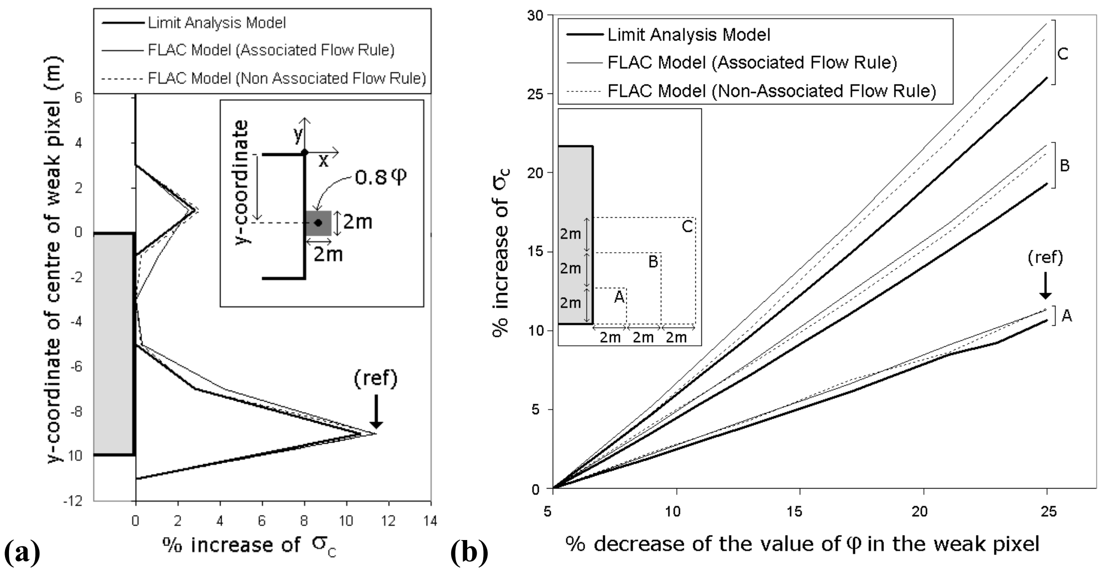


Fig. 5: Impact of (a) square pixel location and (b) square pixel size and percent decrease of φ inside the pixel, on σ_c

Further studies will be necessary to confirm these initial observations and to study other parameters of the weak pixel such as its shape and its orientation. The reasonable agreement between results from the proposed mechanism and those from FLAC model for the above cases involving a local weakness is quite encouraging.

CONCLUSION

This paper presented a new kinematically admissible failure mechanism in the framework of the kinematical approach of limit analysis for determining the critical collapse pressure of a pressurized tunnel face. This mechanism can account for spatial variability of the friction angle of the soil. In this mechanism, the normality condition imposed by limit analysis is enforced everywhere along the slip surfaces of the failure mechanism. The proposed method was validated for purely frictional soils (both homogeneous and heterogeneous) using FLAC. The heterogeneity was introduced via a square pixel that may vary in size and in the value of the decrease in the friction angle. There was a good agreement between limit analysis and FLAC results. Thus, the proposed limit analysis method is promising and worthy of further pursuit given its computational efficiency. Further studies will have to be carried out to validate the proposed approach more extensively for different shapes (e.g. rectangle, layer…), locations and orientations of the pixel and finally in the general case of a soil modelled by a random field.

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