Range of the Safe Retaining Pressures of a Pressurized Tunnel Face by a Probabilistic Approach

Guilhem Mollon¹; Daniel Dias²; and Abdul-Hamid Soubra, M.ASCE³

Abstract: A probabilistic analysis of the face stability of a pressurized tunnel is undertaken in this article. First, two existing velocity fields based on the limit analysis theory are briefly described. They allow one to compute the values of the critical pressures of collapse and blowout of a pressurized tunnel face in cases of both frictional and nonfrictional soils. These models, which have the great advantage of a low computational cost, are validated by comparison with a computationally expensive numerical model. Then, an efficient probabilistic method called collocation-based stochastic response surface methodology (CSRSM) is applied on these velocity fields to perform the uncertainty propagation. This method makes it possible to compute the probability of failure of the tunnel face against both collapse and blowout. In the case of a frictional soil, it appears that the blowout of the face is extremely unlikely and that the collapse is the only probable failure mode. On the contrary, in a purely cohesive soil, it appears that both failure modes are likely to appear and should be considered in the analysis. Finally, this paper presents a discussion concerning the application of the proposed probabilistic method for an economic and safe design of a pressurized shield. **DOI:** 10.1061/(ASCE)GT.1943-5606.0000911. © 2013 American Society of Civil Engineers.

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Introduction

In tunneling projects, the stability of the tunnel face is a major issue. When the excavation is performed using a pressurized shield, this stability is ensured by applying a fluid pressure, called σ_t , to the tunnel face. The fluid may be compressed air, slurry, or the earth excavated by the tunnel. In this paper, a uniform pressure is considered, which may be associated with compressed air stabilization. Instability of the face may occur if the fluid pressure is too low [in this case, the failure mode is called collapse, and the motion of the soil is directed toward the face, as shown in Fig. 1(a)] or if it is too high [in this case, the failure mode is called blowout, and the motion of the soil is directed toward the ground surface, as shown in Fig. 1(b)]. This means that the range of the safe fluid pressures that can be applied to a tunnel face is bounded by a lower value (the critical collapse pressure, called σ_c) and an upper value (the critical blowout pressure, called σ_b). For the sake of simplicity, it is supposed in this article that there is no surcharge loading σ_s on the ground surface.

While the models of prediction of the tunnel face instability become more and more accurate, an important issue for the designers is to deal with the uncertainties related to the input parameters of such models. In engineering practice, this uncertainty is dealt with using

¹Research Associate, Dept. of Civil and Environmental Engineering, Hong Kong Univ. of Science and Technology, Clear Water Bay, Kowloon, Hong Kong (corresponding author). E-mail: guilhem.mollon@gmail.com

²Professor, Laboratoire d'Etude des Transferts en Hydrologie et Environnement, Univ. Joseph Fourier Grenoble 1, 38041 Saint Martin d'Hères, France. Email: d.dias69@gmail.com

³Professor, Dept. of Civil Engineering, Univ. of Nantes, 44600 Saint Nazaire, France. E-mail: abed.soubra@univ-nantes.fr

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a global safety factor. The target safety factor adopted in design is based on engineering judgment and/or design codes. More recent and appealing approaches for a safe design of civil engineering projects in the presence of uncertainties are based on probabilistic methods. Such methods require assessing the propagation of the uncertainties from the input variables to the system output that is used in design. This propagation can be performed by a number of methods, and the main issue is related to the very high computational cost of most of these methods.

Several attempts have been made by the authors of the present paper to perform a probabilistic study of the collapse of a pressurized tunnel face, using simplified deterministic models and/or approximate probabilistic methods such as the first-order reliability method (FORM) or the response surface method (RSM), as described in Mollon et al. (2009a, b). The use of simplified deterministic models and approximate probabilistic methods was related to the high computational costs of the numerical deterministic models and the Monte Carlo probabilistic approach (which are believed to lead to accurate solutions). However, efforts have been made since then by the authors of this paper (1) to build up new deterministic analytical models providing accurate critical pressures with a limited time cost (Mollon et al. 2011b, 2012); and (2) to implement an efficient probabilistic method called collocation-based stochastic response surface methodology (CSRSM), which provides accurate probabilistic results from a limited number of calls of the deterministic models (Mollon et al. 2011a). Hence, this article aims at applying CSRSM to these new deterministic models. Because both the deterministic models and the probabilistic approach have been improved, this study is expected to provide more accurate probabilistic results than the previously published ones with a smaller computation time. Moreover, this paper focuses not only on the face collapse but also on the blowout phenomenon of this face, allowing one to obtain the value of the failure probability, not only against collapse, but against both modes of failure (collapse and blowout). Finally, this paper presents a discussion concerning the application of the proposed probabilistic method for an economic and safe design of a pressurized shield.

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Overview of Existing Deterministic Models

The face stability of circular tunnels driven by pressurized shields has been investigated by several authors in the literature. For the case of a frictional soil, some authors performed experimental tests (Chambon and Corté 1994; Takano et al. 2006). Others (Leca and Dormieux 1990; Anagnostou and Kovari 1996; Mollon et al. 2009a, b, 2010, 2011b) performed analytical or numerical approaches. The model by Leca and Dormieux (1990) is based on the kinematic theorem of limit analysis. This model was successively improved using several kinematically admissible velocity fields (see Mollon et al. 2009a, 2010, 2011b). The collapse and blowout velocity fields by Mollon et al. (2011b) are the most critical ones and will be referred to in this paper as the M1 model. This model is briefly presented in this paper. Concerning the case of a purely cohesive soil, one may cite the investigations by Broms and Bennermark (1967), Davis et al. (1980), Ellstein (1986), Augarde et al. (2003), Osman et al. (2006), and Klar et al. (2007), among others. More recently, Mollon et al. (2012) have presented a velocity field in the framework of the kinematic theorem of limit analysis. This velocity field (referred to in this paper as the M2 model) is an extension of the work by Osman et al. (2006) and Klar et al. (2007). It will also be briefly presented in this paper.

The following subsections aim at briefly presenting (1) a computationally expensive numerical model called the numerical model of reference, which is believed to give somewhat accurate solutions; (2) the analytical model M1 by Mollon et al. (2011b) for frictional soils; (3) the analytical model M2 by Mollon et al. (2012) for purely cohesive soils; and last (4) a validation of the M1 and M2 models (via a comparison of their results with those given by the numerical model of reference) for use in a probabilistic framework.

Numerical Model of Reference

The numerical model is based on numerical simulations using *FLAC3D* software. It is shown in Fig. 2. The model dimensions are 50 $\times 40 \times 26$ m in the transversal, longitudinal, and vertical directions, respectively, for a tunnel diameter D = 10 m and a cover-to-depth ratio C/D = 1. The vertical dimension may be modified to deal with different C/D ratios. The soil is assigned an elastic perfectly plastic constitutive model based on Mohr-Coulomb failure criterion. An



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associated flow rule was used to make the comparison meaningful, but it should be noted that the influence of the dilation angle on the critical collapse and blowout pressure is rather limited (Mollon 2010). The use of the present numerical model in a probabilistic framework is very inconvenient because its computation time is close to 50 h. Indeed, an accurate estimate of σ_c or σ_b requires a very fine mesh (215,000 elements). Details on this issue are provided in Mollon et al. (2011c) in a two-dimensional (2D) framework. Notice however that this numerical model may be used as a reference tool to validate other less costly deterministic models. A purely frictional soil ($\varphi = 30^\circ$, c = 0 kPa) and a purely cohesive soil ($\varphi = 0^{\circ}$, c = 30 kPa) were introduced in the numerical model, and the values of σ_c were computed for both cases, giving, respectively $\sigma_c = 23.7$ kPa and $\sigma_c = 53.1$ kPa. Slightly lower pressures than σ_c were applied to both soils to trigger instability, and the deformations of the mesh in the plane of symmetry of the tunnel are shown in Fig. 3. This figure clearly shows that the collapse phenomenon is very different in frictional and nonfrictional soils. Indeed, the face collapse in a frictional soil involves a rotational motion of a single rigid block. Such a failure exhibits a thin shear band located between the moving block and the soil at rest. On the other



Fig. 3. Deformation of the numerical mesh during a face collapse: (a) $\varphi = 30^{\circ}$ and c = 0 kPa; (b) $\varphi = 0^{\circ}$ and c = 30 kPa

hand, in a purely cohesive soil, one may observe a continuous deformation of the soil mass without any shear band. Based on these observations, it seems that a single analytical model would not be able to provide accurate estimates of the critical pressures for both frictional and nonfrictional soils. Two different limit analysis models based on the kinematic approach are used herein for frictional and nonfrictional soils. These models are described in the next subsections. Notice that because of the use of the kinematic theorem of limit analysis, one obtains a rigorous lower bound of σ_c and a rigorous upper bound of σ_b . The M1 and M2 models therefore provide a rigorous bracketing of the admissible interval for the applied face pressure σ_t for both frictional and nonfrictional soils.

Overview of M1 Analytical Model for Frictional Soils

The deterministic model used for the computation of σ_c and σ_b in the case of a frictional soil (with or without cohesion) will be called M1 in this article. It was extensively presented in Mollon et al. (2011b), and is briefly described hereafter. Two rotational rigid block mechanisms are considered for the active (collapse) and passive (blowout) cases. Fig. 4 shows views of the collapse and blowout mechanisms for $\varphi = 30^{\circ}$ and c = 0 kPa. A cylindrical rotational velocity field is considered for the two mechanisms. This means that both mechanisms rotate with a uniform angular velocity ω around a horizontal axis. The only difference between the collapse and blowout cases is the direction of the rotational motion. The failure mechanisms are entirely defined by the position of the axis of rotation, and therefore depend on only two parameters. This is because the envelope of the moving block is constructed point by point in such a manner to respect the normality condition imposed by limit analysis at the different points along the velocity discontinuity surface, and such that its intersection with the tunnel face corresponds to the circular contour of this face. Notice that in the blowout case, the failure mechanism outcrops at the ground surface [Fig. 4(b)]. In the collapse case, it may be the same, but the most usual case corresponds to a mechanism which closes to itself without reaching the ground surface [Fig. 4(a)]. For both mechanisms, the computation time is nearly equal to 3 min using the four processors of a Intel Core 2 Quad CPU 2.4 GHz computer (Santa Clara, California).

Overview of M2 Analytical Model for Purely Cohesive Soils

The numerical results of Fig. 3 have shown that failure mechanisms involving the motion of rigid blocks are not relevant for the failure of a tunnel face in a purely cohesive soil. To explore suitable velocity fields, one has to consider failure patterns involving continuous deformation of the soil mass between the tunnel face and the ground surface with no velocity jump at the envelope of the velocity field. This is the aim of the M2 analytical model. This model was developed in Mollon et al. (2012) and is briefly described hereafter in the collapse case; the blowout case being straightforward. As shown in Fig. 5, the external envelope of the velocity field appears similar to a torus of center O, but it should be noted that this is not an actual torus [more details are provided in Mollon et al. (2012)]. The soil velocity during failure is zero on this envelope, and follows an asymmetric parabolic profile inside it.

The normality condition in a purely cohesive soil implies that any plastic deformation may occur without any change of volume. This assumption is used herein to compute the components of the velocity field everywhere in the somewhat-toric domain. The computation of a critical collapse or blowout pressure by the M2 model takes approximately 20 s, using the four processors of a 2.4 GHz quad-core CPU computer.

Numerical Validation of M1 and M2 Analytical Models

The numerical model introduced in *FLAC3D* and shown in Fig. 2 is used to evaluate the accuracy of the M1 and M2 analytical models. As stated earlier in this article, the M1 and M2 models developed in



Fig. 4. Layout of the face failure as predicted by the M1 model for the case $\varphi = 30^\circ$, c = 0 kPa, and C/D = 1: (a) collapse; (b) blowout



Fig. 5. Layout of the 3D envelope of the M2 model in case of collapse or blowout

the framework of the kinematic theorem of the limit analysis theory provide rigorous lower bounds of σ_c and rigorous upper bounds of σ_b . To be accurate, a limit analysis model should provide a value of σ_c (respectively, σ_b) as high (respectively, as low) as possible. Tables 1 and 2 provide a comparison between the values of σ_c and σ_b provided by the present kinematical models and those given by the

Table 1. Critical Collapse and Blowout Pressures as Obtained by the M1 Analytical Model and the *FLAC3D* Numerical Model for D = 10 m and $\gamma = 18 \text{ kN/m}^3$

		$\varphi = 17^\circ$; $c = 7 \text{ kPa}$				$\varphi = 25^\circ$; $c = 10 \text{ kPa}$			
	Collapse pressure (kPa)		Blowout pressure (kPa)		Collapse pressure (kPa)		Blowout pressure (kPa)		
C/D	M1	FLAC3D	M1	FLAC3D	M1	FLAC3D	M1	FLAC3D	
0.6	37.8	36.8	682.4	635	13.9	11.7	1,112.2	1,091	
0.8	37.8	37.2	878.6	864	13.9	12.1	1,487.3	1,521	
1	37.8	37.3	1,096.6	1,113	13.9	12.2	1,903.5	2,004	
1.5	37.8	37.3	1,777.1	1,842	13.9	12.2	3,301.2	3,488	
2	37.8	37.4	2,637.7	2,740	13.9	12.2	5,213.3	5,337	
3	37.8	37.4	5,243.1	5,253	13.9	12.2	_	—	

Table 2. Critical Collapse and Blowout Pressures as Obtained by the M2 Analytical Model and by the *FLAC3D* Numerical Model for D = 10 m and $\gamma = 18$ kN/m³

		$c_u = 2$	20 kPa		$c_u = 30 \mathrm{kPa}$			
	Collapse pressure (kPa)		Blowout pressure (kPa)		Collapse pressure (kPa)		Blowout pressure (kPa)	
C/D	M2	FLAC3D	M2	FLAC3D	M2	FLAC3D	M2	FLAC3D
0.6	84.8	86.7	316.0	289	20.2	22.1	386.2	358
0.8	106.5	108.8	388.5	342	34.6	37.5	473.3	419
1	129.4	132.9	443.9	392	50.7	53.1	538.2	474
1.5	192.1	199.1	565.0	509	97.8	106.3	675.9	604
2	256.9	272.7	683.2	617	150.5	169.5	808.5	726
3	398.9	447.9	906.0	814	272.1	340.7	1,054.0	922

numerical *FLAC3D* model. In the case of frictional soils, Table 1 shows that the M1 model provides solutions that are not far from the values provided by the numerical model. Similar agreement was obtained for several cases of frictional soils with or without cohesion (not shown in this paper). Furthermore, as was mentioned in Mollon et al. (2011b), the present velocity fields provide the best bounds of the critical pressures (that is, the highest values of σ_c and the lowest values of σ_b). The same conclusions found for the M1 model can be drawn for the M2 model. The M1 and M2 models can therefore be applied with confidence in a probabilistic context instead of the numerical model, with a limited loss of accuracy but for a much lower computational cost.

Probabilistic Study

This section aims at investigating the effect of the uncertainties related to the soil properties and the applied face pressure on the tunnel face stability. Six soil configurations are considered: (1) two purely frictional soils corresponding to sands (called S1 and S2); (2) two frictional and cohesive soils corresponding to drained clays (called DC1 and DC2); and (3) two purely cohesive soils corresponding to undrained clays (called UC1 and UC2). The mean values of the soil shear strength parameters, the soil unit weight, the tunnel diameter and the tunnel cover depth corresponding to each configuration are summarized in Table 3. This table provides the deterministic values of σ_c and σ_b for the six configurations as provided by the models M1 (for S1, S2, DC1, and DC2) and M2 (for UC1 and UC2).

In the following, one first presents the uncertain input parameters used in the analysis. This is followed by the method used for the propagation of the uncertainties from the input data to the system responses (σ_c and σ_b). Then, the failure probability against both collapse and blowout of the tunnel face is computed. Finally, a design based on a target failure probability is proposed.

Uncertain Input Data

Five uncertain parameters (see Table 4) were considered in the analysis. Three probabilistic scenarios were studied because the variability of the input random variables is generally not known with accuracy. The neutral scenario makes use of some classical values of the coefficients of variation (COVs) of the random variables, while the optimistic and pessimistic scenarios deal with smaller and larger values of the COVs, respectively. All the values of the COVs are provided in Table 4. It should be mentioned that the variability of the soil shear strength parameters (φ , c) is quite well documented (Phoon and Kulhawy 1999). This is related to the natural variability of the soil and to the possible measurement errors. The unit weight is a soil characteristic that is easy to assess, which explains the low values of COV for this variable. In the same way, the cover depth C is controlled by the operator of the tunnel boring machine (TBM), and is subject to a low level of uncertainty. The COV of the fluid pressure σ_t applied at the tunnel face is very low (5%) in the optimistic scenario, and quite high (25%) in the pessimistic one, depending on the level of control of this pressure (related to the tunnel machinery and human calibration). This variability also accounts for the uncertainty on the soil behind this pressure, especially its permeability to the retaining fluid.

In the present paper, each input random variable will be modeled using two different probability density functions (PDFs). In the socalled normal variables case, the five variables are considered as Gaussian, while in the so-called nonnormal variables case, the soil friction angle φ is assumed to follow a beta distribution (bounded by 0 and 45°) and the four other variables are assumed to follow

Table 3. Soil and Geometrical Parameters Used in the Six Configurations

 Considered in the Analysis and Their Corresponding Collapse and Blowout

 Pressures

					Deterministic results		
Soil	μ_{φ} (degrees)	μ_c (kPa)	$D.\mu_{\gamma} (kN/m^2)$	$\mu_{\rm C}/{ m D}$	Model	σ_c (kPa)	σ_b
S 1	40	_	150	1	M1	12.4	2,984
S2	25	N/A	250	1	M1	49.0	2,333
DC1	17	7	180	1	M1	37.8	1,097
DC2	20	20	250	1	M1	13.6	2,056
UC1		20	150	1	M2	81.6	401.4
UC2	N/A	30	250	1.5	M2	244.1	810.5

Table 4. Description of the Probabilistic Scenarios for the Five Random Variables

Variable	Optimistic scenario	Neutral scenario	Pessimistic scenario	Type of distribution
Internal friction angle φ (degrees)	5	10	15	Gaussian or Beta
Cohesion c (kPa)	10	20	30	Gaussian or Lognormal
Unit weight $\gamma (kN/m^3)$	3	5	8	Gaussian or Lognormal
Cover depth (m)	1	3	5	Gaussian or Lognormal
Applied face pressure σ_t (kPa)	5	15	25	Gaussian or Lognormal

a lognormal distribution. A possible correlation may exist between φ and c, although there is no large consensus in the literature on the value of this correlation. In the present article, when dealing with frictional and cohesive soils (DC1 and DC2), it is chosen to use correlation values of -0.4, 0, and +0.4 in the optimistic, neutral, and pessimistic scenarios, respectively.

Method Used for Uncertainty Propagation

As is well known, the most robust and accurate probabilistic approach is the Monte Carlo simulation method. This method makes possible the propagation of the uncertainties from the input data to the model output through a deterministic model to compute the PDF of the system response. However, this method requires a large number of calls of the deterministic model (about 1 million samples for a target failure probability of 10^{-5}). This number of calls is obviously out of reach for most deterministic models because of their computational costs. To overcome this shortcoming, a powerful probabilistic method called CSRSM is used in this paper. The principles of this method are briefly described herein. The reader may find a more detailed description in Isukapalli (1999), Sudret (2008), Phoon and Huang (2007), Huang et al. (2009), Mollon et al. (2011a), and Li et al. (2011).

The CSRSM allows one to obtain an analytical approximation (called a meta-model) of a random system response. Thus, the system response may be computed more quickly when using any classical probabilistic method (such as Monte Carlo sampling). This is because one uses the meta-model and not the original deterministic model. In the framework of CSRSM, the meta-model is a polynomial chaos expansion (PCE) of a given order n. In this paper, it is expressed in the basis of the multidimensional Hermite polynomials of orders $\leq n$, although other types of polynomials may also be used (cf. Xiu and Karniadakis 2002). For a given set of the probabilistic parameters of the input random variables, the unknown coefficients of the PCE are obtained hereafter by regression using the response of the original deterministic model at a given number of so-called collocation points. This number is much smaller than the number of simulations required by the crude Monte Carlo simulation methodology applied on the original deterministic model. As proposed in Mollon et al. (2011a), PCEs of order 4 are used in this study. Different number of random variables are used for the different soil configurations (see Table 3): (1) three random variables for S1 and S2 (these variables are φ , γ , and C); (2) three random variables for UC1 and UC2 (c, γ , and C); and (3) four random variables for DC1 and DC2 (φ , c, γ , and C). A very detailed description involving the application of CSRSM to the face collapse was proposed in Mollon et al. (2011a). This study was not expected to provide accurate results because it used a simple and biased deterministic model and considered a limited number of random variables. Thus, the paper by Mollon et al. (2011a) mainly had a methodological value, and the readers are expected to find in that paper the theoretical background of CSRSM to reproduce the present analysis. For this reason, the present paper does not provide significant details on the application of CSRSM to the proposed deterministic models and focuses only on the results.

Fig. 6 presents the PDFs of σ_c and σ_b in the case of the soil configuration DC1, for the three probabilistic scenarios and for the two types of variables (similar trends were obtained for the five other configurations). These PDFs are obtained by Monte Carlo simulations (1 million samples) performed on the meta-models obtained after the application of CSRSM. Fig. 6 emphasizes the important influence of the probabilistic scenario on the PDFs of the critical pressures. Indeed, an optimistic scenario leads to rather narrow distributions of the critical pressures, while a pessimistic scenario leads to much more spread-out distributions and thus to a larger variability. On the contrary, the influence of the type of variables seems rather limited. This is

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Fig. 6. Probability density functions (PDFs) of the critical collapse and blowout pressures for the soil configuration DC1

confirmed by the results presented in Table 5, which provides the COVs of σ_c and σ_b for the six soil configurations, for the three probabilistic scenarios, and for the two types of variables. This table shows that the COVs can have very different values, running from 7.0 to 124.4% for σ_c and from 4.7 to 49.6% for σ_b .

Failure Probability against Collapse and Blowout

It would be desirable to assess for a chosen face pressure $\mu_{\sigma t}$ the probability of failure against a single mode of failure (collapse or blowout) of the tunnel face, or more importantly the probability of failure against both collapse and blowout. The failure probability against collapse (respectively, blowout) corresponds, respectively, to the event $G_1 < 0$ (respectively, $G_2 < 0$) where

$$G_1 = \sigma_t - \sigma_c \tag{1}$$

$$G_2 = \sigma_b - \sigma_t \tag{2}$$

The global failure probability against collapse and blowout corresponds to the event ($G_1 < 0$ or $G_2 < 0$). For a given probabilistic scenario and for a given type of random variables (normal or nonnormal), it is possible to run Monte Carlo simulations and evaluate the probability of failure against collapse or/and blowout very quickly. A number of samples of 1 million is chosen to achieve a satisfactory level of accuracy for a target failure probability as low as 10^{-5} . Notice that for each sample, the value of σ_t is obtained directly by generating a random value from the PDF of σ_t , because this PDF is known and no uncertainty propagation is needed; however, the value of σ_c (or/and σ_b) is obtained by generating a vector of standard random variables (whose dimension is equal to the number of the uncertain parameters) and by replacing this vector into the meta-model of σ_c (and/or σ_b) to compute the value of G_1 (and/or G_2) using Eqs. (1) and (2) at a quasi-null time cost.

The failure probability against a single mode of failure (collapse or blowout) is provided in Figs. 7 and 8, respectively, for the sands (S1 and S2) and for the drained clays (DC1 and DC2). In each of these

Table 5. Coefficients of Variation of σ_c and σ_b for the Six Soil Configurations for (1) the Three Probabilistic Scenarios and (2) the Two types of PDF

	Optimis	tic scenario	Neutra	l scenario	Pessimistic scenario	
Case	Normal variables	Nonnormal variables	Normal variables	Nonnormal variables	Normal variables	Nonnormal variables
S1						
σ_c	11.6	11.8	23.0	24.1	34.8	37.8
σ_b	15.0	14.8	30.5	30.9	46.7	49.6
S2						
σ_c	7.0	8.5	15.9	16.6	24.3	25.3
σ_b	7.5	7.5	15.1	15.3	22.9	23.6
DC1						
σ_{c}	8.9	8.8	20.0	19.9	35.3	33.9
σ_b	6.4	6.4	12.6	12.8	21.3	21.7
DC2						
σ_c	35.4	34.6	83.6	82.0	124.4	116.2
σ_b	6.7	6.8	13.8	13.9	23.6	23.9
UC1						
σ_c	21.2	21.2	41.3	41.0	62.7	61.2
σ_b	4.8	4.7	9.9	9.8	15.0	14.6
UC2						
σ_{c}	13.2	13.2	25.6	25.4	39.0	38.1
σ_b	4.7	4.7	8.9	8.8	13.5	13.2

figures, the curves on the left-hand side correspond to a collapse of the face, and the curves on the right-hand side correspond to a blowout. The failure probabilities are very high for low values and high values of the applied face pressure $\mu_{\sigma t}$ corresponding, respectively, to collapse and blowout of the soil. In the central part of these figures, one may observe an interval of the $\mu_{\sigma t}$ value for which both failures by collapse and blowout have probabilities of occurrence smaller than 10^{-5} . Such an interval can therefore be considered as the range of the safe values of the applied face pressure $\mu_{\sigma t}$. The value of 10^{-5} was chosen here as an example, but a different acceptable probability of failure may be chosen depending on the criticality of the existing buildings at the ground surface and/or on local regulations. Figs. 7 and



Fig. 7. Failure probability against a single mode of failure (collapse for the left-hand curves and blowout for the right-hand curves) versus the applied face pressure $\mu_{\sigma t}$ for the soil configurations S1 and S2

8 show that, for frictional soils (with or without cohesion), the width of this interval is highly dependant on the soil configuration and on the probabilistic scenario. An optimistic scenario leads to a very wide safe interval, while a pessimistic scenario strongly reduces its width. The type of variables (normal or nonnormal) may also have an influence on this interval, but no general trend appears. For the soil configuration DC1 (in the case of a pessimistic scenario and normal variables), the safe interval is rather narrow (from 250 to 350 kPa).

The two configurations corresponding to purely cohesive soils (UC1 and UC2) show a different behavior. Indeed, Fig. 9(a) shows

an example of this behavior for the UC1 soil in the case of normal variables and a neutral scenario. It appears that the safety interval pointed out in the case of frictional soils does not exist anymore because the curves of failure probability against collapse and blowout meet before reaching a value smaller than 10^{-5} . This is mostly related to the fact that the blowout in purely cohesive soils occurs at much smaller values of the face pressure than in frictional soils (Table 3). For this reason, it appears that there is no value of μ_{ot} for which both failure probabilities would be lower than 10^{-5} . Because the two failure modes cannot occur simultaneously, it is



Fig. 8. Failure probability against a single mode of failure (collapse for the left-hand curves and blowout for the right-hand curves) versus the applied face pressure $\mu_{\sigma t}$ for the soil configurations DC1 and DC2

possible to define a global failure probability by adding the values of the failure probability of the two modes. This operation is performed in Fig. 9(b) and leads to a single curve of the failure probability with a minimum value. Notice that the values of the global failure probability could also be obtained by applying Monte Carlo simulation methodology on the meta-model under the condition ($G_1 < 0$ or $G_2 < 0$). It should be emphasized here that the value of the applied tunnel pressure corresponding to the minimum failure probability is that for which neither modes or failure (collapse or blowout) is predominant; however, smaller and greater values of the applied tunnel pressure lead to greater values of the global failure probability because of the predominance of the collapse mode of failure for the smaller values of the applied pressure (the effect of blowout being less significant in this case) and the predominance of the blowout mode of failure for the larger values of this applied pressure (the effect of collapse being less significant in this case).

Fig. 10 presents similar curves to Fig. 9(b), but for the three probabilistic scenarios, for the two types of variables, and for the two soil configurations UC1 and UC2. Once again, it appears that the probabilistic scenario has an important influence on the failure



Fig. 9. (a) Failure probability against single mode of failure (collapse or blowout) versus the applied face pressure $\mu_{\sigma t}$ for the UC1 soil; (b) failure probability against both mode of failure (collapse or blowout) versus the applied face pressure $\mu_{\sigma t}$ for the same soil

probability. In the case of an optimistic scenario, one may observe a safety interval (for which the failure probability is lower than 10^{-5}) similar to the one observed for frictional soils. However, in the neutral and pessimistic scenarios, it is not possible to achieve a failure probability lower than 10^{-5} . In the case of normal variables and for the soils UC1 and UC2, respectively, the lowest failure probabilities that one may reach are equal to 0.0018 and 0.0061 in a neutral scenario and to 0.047 and 0.082 in a pessimistic scenario. These values are surprisingly high, especially when one considers that they are minimum values and that any other choice of $\mu_{\sigma t}$ can only increase them. The use of nonnormal random variables seems to reduce both the minimum value of the failure probability and the corresponding value of $\mu_{\alpha t}$, but this cannot be considered as certain based on this limited number of cases. One should also notice that the value of the minimum failure probability is not linked to the width of the deterministic range of safety (that is, the interval between the deterministic values of σ_c and σ_b). Indeed, this interval is much larger for UC2 than for UC1 (see Table 3), while the maximum level of safety is larger for UC1 than for UC2.

As a first attempt to find some general trends on the safety of pressurized tunnels in purely cohesive soils, several $\mu_{\sigma f}$ -P_f curves

are plotted in Fig. 11 for eight purely cohesive soils. These eight cases cover several values of the soil cohesion and unit weight and several values of the tunnel diameter and cover depth in the case of normal variables and for the neutral probabilistic scenario. Fig. 11 shows that the mean value of the cohesion μ_c does not modify the optimal value of $\mu_{\sigma t}$ but highly influences the corresponding value of the failure probability. The mean value of the cover depth ($\mu_{C/D}$) seems to have an influence on the optimal value of $\mu_{\sigma t}$ (larger cover depth leading to larger optimal pressures), but a rather limited impact on the corresponding failure probability. Finally, an increase in the mean value of the soil unit weight (μ_{γ}) leads to higher values of the optimal prescribed pressure, but also to an increase of the corresponding failure probability. Based on these observations, it would appear that the optimum applied pressure evolves in a manner similar to the existing at-rest pressure before excavation (that is, independent from c, but increasing with C/D and γ).

Probabilistic Design

The different theoretical approaches presented in this paper (the deterministic M1 and M2 models and the probabilistic method CSRSM) were gathered in a common platform to create the *Tunnel*



Fig. 10. Failure probability against both modes of failure (collapse and blowout) versus the applied face pressure $\mu_{\sigma t}$ for the soil configurations UC1 and UC2

Face Stability Software (TFSS) program. This software operates in a *MATLAB* environment, and is available for free download at the website, http://guilhem.mollon.free.fr. It is also given in the *Journal* website. The motivation for releasing this program is that it does not seem possible to provide a general and complete chart for the probabilistic design of the retaining face pressure. Indeed, the very large number of parameters of the problem (the tunnel geometry, the statistical characteristics of the random variables, means, COV, types of PDF, correlations between variables, and so on) prevents the plotting of such a design chart. On the other hand, it would be useful to employ this program as a preliminary design tool to identify the

most influential parameters that have a significant contribution in the variability of the system response. This has the great advantage of allowing one to undertake an intensive experimental campaign on only the most influential parameters, which may lead to an economic design, as will be shown later in this section.

To demonstrate the capabilities of the *TFSS* program, two examples (called Example 1 and Example 2) are proposed. They are described in Table 6. Example 1 corresponds to a frictional and cohesive soil, while Example 2 corresponds to a purely cohesive soil. Illustrative probabilistic scenarios were chosen with four random variables for each example. These scenarios (mean values, COV, and type of PDF of the



Fig. 11. Global probability of failure against both modes of failure (collapse and blowout) versus the prescribed face pressure $\mu_{\sigma t}$ for several purely cohesive soils in the case of normal variables and a neutral scenario

Table	6.	Design	Examples
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Parameter	Example 1	Example 2	Example 2 (updated)
Diameter D	8 m	11 m	Unchanged
Cover depth C	12 m	Mean value: 14 m	Unchanged
		COV: 5%	
		Lognormal distribution	
Unit weight γ	Mean value: 20 kN/m^3	Mean value: 22 kN/m^3	Unchanged
	COV: 3%	COV: 5%	
	Lognormal distribution	Lognormal distribution	
Friction angle φ	Mean value: 21°	0°	Unchanged
	COV: 12%		
	Beta distribution		
Cohesion c	Mean value: 5 kPa	Mean value: 38 kPa	Mean value: 34 kPa
	COV: 18%	COV: 25%	COV: 10%
	Lognormal distribution	Lognormal distribution	Lognormal distribution
Correlation between φ and c	-0.3	—	—
Applied face pressure σ_t	COV: 10%	COV: 15%	Unchanged
	Lognormal distribution	Lognormal distribution	

random variables) as well as the problem geometry and some additional tuning parameters (such as the discretization parameters of the deterministic models, the order of the PCE used in CSRSM, or the number of samples used in the Monte Carlo samplings) were introduced in the *TFSS* program.

Because Example 1 corresponds to a tunnel excavated in a frictional soil, the failure by blowout was ignored and only collapse was considered. Indeed, preliminary computations performed by *TFSS* (using the M1 model) showed that the deterministic values of the collapse and blowout pressures were 27 and 1,993 kPa, indicating that the latter failure mode may be considered as unlikely because this value of applied pressure is out of reach of most pressurized shields. Some of the probabilistic outputs of the program are provided in Fig. 12. Fig. 12(a) presents the probability density function of the critical collapse pressure based on its approximation by a PCE of order 4. Monte Carlo samplings (with 10⁶ samples) were then automatically performed for several values of the prescribed face pressure $\mu_{\sigma t}$ to plot the curve of Fig. 12(b), which shows the probability of collapse versus this prescribed face pressure. From this figure, it is very easy to choose a desired level of safety (such as the common value of the reliability index $\beta_{\rm HL} = 3.8$ prescribed by Eurocode 7, which corresponds to a probability of failure of 7.23×10^{-5}) and to determine the corresponding necessary face pressure. As shown in Fig. 12(b), this pressure should be $\mu_{\sigma t} = 63$ kPa in Example 1.

Fig. 13(a) shows similar curves for Example 2, except that both collapse and blowout failure modes were considered in the analysis



Fig. 12. Design example 1: (a) PDF of the critical collapse pressure; (b) failure probability against collapse versus the prescribed tunnel pressure



Fig. 13. Design example 2: (a) failure probability in the initial scenario; (b) failure probability in the updated scenario

because this example deals with a purely cohesive soil. This figure shows that, in the probabilistic scenario defined for Example 2, the global probability of failure (defined as the sum of the probabilities of collapse and blowout) cannot reach a satisfactory value. More specifically, there is no value of the prescribed face pressure $\mu_{\sigma t}$ for which the probability of failure is smaller than the prescribed value of 7.23 × 10⁻⁵. If this example was a real case of excavation, the designer looking for an appropriate face pressure would thus be facing a problem that does not seem to have a solution. However, one should note that the primary reason for this observation is the high level of uncertainty of the input variables. Thus, to achieve a design with the desired level of safety, it would be necessary to gather more information only on the input random variables to decrease their level of uncertainty. The concept of the Sobol's indices was explained in detail in Mollon et al. (2011a), and is very useful for this purpose. These indices provide the weight of the different uncertain parameters in the variability of the system response (in our case, the critical collapse or blowout pressure). The *TFSS* program provides directly the Sobol's indices of each variable as an output, and showed that, in the present case, the variability of the cohesion was

actually responsible for 91.8% (respectively, 90.7%) of the variability of the critical collapse (respectively, blowout) pressure. This information is very valuable for a designer because it clearly indicates the most influential parameters that should be looked for thoroughly to reduce their level of uncertainty. The COV of the cohesion in Example 2 was 25%, for a mean value of 38 kPa. Let us imagine that, based on the preceding results, the designer would chose to drive a more intensive campaign of soil recognition to increase his/her knowledge about the soil cohesion, and would obtain a corrected mean value of 34 kPa with a much smaller COV of 10%. Such updated values, when introduced in TFSS, would lead to the curves of Fig. 13(b), which are much more satisfactory. Indeed, despite the fact that the mean value of the cohesion has been found smaller than its first estimate, the reduction of its COV (that is, the improvement of the designer's knowledge) leads to a much lower value of the failure probability, and makes it possible to reach the desired level of safety of $\beta_{HL} = 3.8$. As shown in Fig. 13(b), a proper design value for this updated scenario would be $\mu_{\sigma t} = 395$ kPa. This example reveals to which extent the probabilistic methods may be used to improve the safety of the pressurized tunnels with a limited economic cost.

Conclusion

This article presents a probabilistic analysis for the determination of the admissible range of the retaining pressure that can be applied to a pressurized tunnel face without it to collapse or blowout. Two deterministic models developed by the authors in previous studies were considered for frictional and nonfrictional soils. Each model is able to provide the critical collapse pressure σ_c and the critical blowout pressure σ_b and, thus, the range of the admissible pressures with a satisfying accuracy and a limited time cost when compared with the classical numerical models.

Three illustrative probabilistic scenarios were chosen to model the uncertainty of the soil parameters and that of the excavation process. More precisely, these scenarios were used to account for the amount of information that is available to the designer and that is related to (1) the quality and the quantity of soil recognition and (2) the excavation monitoring (although a quantification of this relation is a broad topic that is not tackled in the current study). An efficient probabilistic method called CSRSM was used to assess the uncertainty propagation through the deterministic models. Six illustrative configurations that cover a wide range of situations were studied using this method. CSRSM makes it possible (for a given soil configuration and a given probabilistic scenario) to compute efficiently and accurately the probability of collapse and blowout of the tunnel face. Also, a probabilistic design is possible using this method. For this purpose, a MATLAB software called TFSS that makes use of CSRSM and the different deterministic models was provided and illustrated in this paper for practical use in geotechnical engineering. The main findings of this paper can be summarized as follows:

The probabilistic scenario has a very large influence on the COV of the critical pressures: an optimistic scenario leads to a small variability of these pressures, while a pessimistic scenario increases by a lot their dispersion. It is thus desirable in a practical context to tend as much as possible toward an optimistic scenario—that is, to perform the necessary soil recognitions and excavation monitoring aiming at reducing the level of uncertainty. The computer program provided in this paper should help in identifying the most influential parameters that need further experimental investigation to lead to a safe design with a reduced economic cost.

- The failure probability is very high for low values and high values of the applied face pressure $\mu_{\sigma t}$ corresponding, respectively, to collapse and blowout. This probability is smaller for intermediate values of the applied pressure. For these smaller values of the failure probability, one can define a zone of desirable applied pressures corresponding to a target failure probability.
- In the case of frictional soils (with or without cohesion), there is an interval for $\mu_{\sigma t}$ for which the failure probability against collapse and the failure probability against blowout are smaller than 10^{-5} . This range can be considered as a safe interval. An optimistic scenario leads to a very wide safety interval, while a pessimistic scenario reduces the width of this interval. It appears that the failure by blowout is very unlikely in frictional soils and that the design may be performed solely with respect to collapse.
- In nonfrictional soils, the safety interval pointed out in the case of frictional soils no longer exists, and there is no value of the prescribed pressure $\mu_{\sigma t}$ for which the failure probability against collapse and the failure probability against blowout are smaller than 10^{-5} . Thus, a proper design of the face-retaining pressure in such soils requires considering both modes, and for some probabilistic scenarios it may not be possible to reach the desired level of safety. An extensive experimental campaign may thus be necessary on the most influential parameters to lead to a safe and economic design.

The main drawback of probabilistic methods is that the statistical distributions of the soil parameters (and more generally of all the uncertain parameters involved in an excavation) are never known with a good precision. Thus, it is very difficult for a designer to assess in which probabilistic scenario the design should be performed. It would be desirable in future works to build up tools to determine the level of uncertainty related to the soil properties in different contexts, in relation with the quantity and quality of the soil recognitions. If such a tool is developed in the future, the current study provides a rigorous methodology that will make it possible to perform accurate and reliable designs of the face-retaining pressure of a pressurized tunnel.

Supplemental Data

The fully working *TFSS* code (running in a *MATLAB* environment) and a text file containing a short user guide are available online in the ASCE Library (www.ascelibrary.org).

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